

# The Nonperturbative Wave Functions, Transverse Momentum Distribution and QCD Vacuum Structure.

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## Abstract

It is shown that there is one-to-one correspondence between two, apparently different problems:

1. The determination of the meanvalues of transverse moments  $\langle \vec{k}_\perp^{2n} \rangle$  for the nonperturbative pion wave function  $\psi(\vec{k}_\perp^2, x)$  and
2. The evaluation of the mixed vacuum condensates  $\langle \bar{q} G_{\mu\nu}^n q \rangle$ .

Arguments in favor of a large magnitude of the mixed vacuum condensate  $\langle \bar{q}(ig\sigma_{\mu\nu}G_{\mu\nu})^2q \rangle$  are given. The analysis is based on the dispersion relations and PCAC. Because of the large values of the condensates we found a noticeable fluctuations of the momentum  $\langle \vec{k}_\perp^4 \rangle > \langle \vec{k}_\perp^2 \rangle^2$ . We also found some general properties of the condensates  $\langle \bar{q}(ig\sigma_{\mu\nu}G_{\mu\nu})^nq \rangle$  for arbitrary  $n$ . This information is used for the analysis of the higher moments  $\langle \vec{k}_\perp^{2n} \rangle$  in the limit when the space-time dimension  $d \rightarrow \infty$ . As a byproduct, it is proven that the standard assumption on factorizability of the  $\psi(\vec{k}_\perp^2, x) = \psi(\vec{k}_\perp^2)\phi(x)$  **does contradict** to the very general properties of the theory. We define and model  $\psi(\vec{k}_\perp^2, x)$ , satisfying all these constraints.

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## 1. Introduction

The problem of bound states in the relativistic quantum field theory with large coupling constant is an extremely difficult problem. To understand the structure of the bound state is a very ambitious goal which assumes the solution of a whole spectrum of tightly connected problems, such as confinement, chiral symmetry breaking phenomenon, and many, many others which are greatly important in the low energy region.

A less ambitious purpose is the study of the hadron wave function ( $wf$ ) with a minimal number of constituents. As is known such a function gives parametrically leading contributions to hard exclusive processes. In this case the quark and antiquark are produced at small distances  $z \sim 1/Q \rightarrow 0$ , where  $Q$  is the typical large momentum transfer. Thus, we can neglect the  $z^2$  dependence of the wave function of the meson with momentum  $p$  and can concentrate on the variable  $zp \simeq 1$  which is order of one. Therefore, the problem is drastically simplified in the asymptotic limit and we end up with the light cone wave function,  $\phi(zp, z^2 = 0)$ .

The corresponding wave functions have been introduced to the theory in the late seventies and early eighties [1] in order to describe the exclusive processes in QCD. We refer to the review papers [2],[3], [4] on this subject for the detail definitions and discussions in the given context, but here we want to make a short remark concerning the very unusual properties of the non-perturbative light-cone wave functions of the leading twist. The information which can be extracted from the QCD sum rules method [5],[6] unambiguously shows the asymmetric form of the distribution amplitudes and this property was unexpected and even suspicious to many physicists.

However, to study the fine aspects of the theory (see explanations below) of the exclusive processes and to extend the area of applications, we need to know not only the dependence  $wf$  on the longitudinal variable  $x_i$ , but on transverse variable  $\vec{k}_\perp^2$  as well. In particular, as is known [7],[8], the Sudakov suppression should be taken into account in order to integrate correctly over the “endpoint region” ( $x_i \rightarrow 1$ ). The dependence of  $wf$  on the transverse momentum  $\vec{k}_\perp^2$  plays an important role in such calculation. Besides that, the transverse size dependence plays a key role in color transparency physics.

- The main goal of the paper is the calculation of the few lowest moments  $\langle \vec{k}_\perp^{2n} \rangle$  of the transverse distribution. Besides that we formulate the definition of nonperturbative wave function and find some constraints on it. In particular, we analyze the asymptotically distant terms  $\langle \vec{k}_\perp^{2n} \rangle$ ,  $n \rightarrow \infty$  in order to model the  $\psi(\vec{k}_\perp^2, x_i)$ . The byproduct of this consideration is analysis of the factorizability of  $\psi(\vec{k}_\perp^2, x_i) = \phi(x_i)\psi(\vec{k}_\perp^2)$ , which it turns out, does not work. Finally, we model  $wf$  which meets all constraints. Analogous problem for the longitudinal distribution  $\langle \xi^n \rangle$  has been formulated and solved within QCD sum rules method in ref.[9].

The paper is organized as follows. In the next section we derive the explicit relations between two, apparently different, values:

1.  $\langle \vec{k}_\perp^{2n} \rangle_P$  for pseudoscalar  $wf$ ,  $\phi_P = \langle 0 | \bar{d}(z) i\gamma_5 u(-z) | \pi \rangle$  on the one side and  
 2. vacuum expectation values (VEVs)  $\langle \bar{q} G_{\mu\nu}^n q \rangle$  on the other side. Such exact (in chiral limit) connection is not very surprising because  $\pi$  meson is the Goldstone particle, strongly interacting with vacuum fluctuations. Well-known example of such connection is, of course, the formula

$$\langle 0 | \bar{d} i\gamma_5 u | \pi \rangle = -\frac{1}{f_\pi} \langle 0 | \bar{d} d + \bar{u} u | 0 \rangle, \quad (1)$$

which relates  $\pi$  meson matrix element and some vacuum characteristic.

In the section 3 we derive the analogous relations for  $\langle \vec{k}_\perp^{2n} \rangle_A$  for axial (leading twist)  $wf$ . In this case the corresponding formulae are not exact even in the chiral limit, but we argue that their accuracy is very high and corrections are of order  $\alpha_s/\pi \simeq 10\%$ . We will see that for both cases (axial and pseudoscalar) the problem is reduced to the evaluation of the mixed vacuum condensates  $\langle \bar{q} G_{\mu\nu}^n q \rangle$ .

In order to estimate them we consider in section 4 some special kind of sum rules which are very sensitive to the VEVs we are interested in. We find a large violation (order of factor 3) of factorization for mixed condensates  $\langle \bar{q} (\sigma_{\mu\nu} G_{\mu\nu})^2 q \rangle$  of dimension seven. It automatically leads to a noticeable fluctuations of the momentum  $\langle \vec{k}_\perp^4 \rangle_A > \langle \vec{k}_\perp^2 \rangle_A^2$  and leads to the well-spread out  $wf$  (in the momentum space). Let us note, that if the factorization would work, we would get  $\langle \vec{k}_\perp^4 \rangle_{A,P} \approx \langle \vec{k}_\perp^2 \rangle_{P,A}^2$ . Such a relation gives the *qualitatively* different behavior for  $wf$  and means a strong concentration of the distribution density around point  $\vec{k}_\perp^2 = 0$ .

In the section we derive an “almost” exact relation between mixed vacuum condensates  $\langle \bar{q} (ig\sigma_{\lambda\sigma} G_{\lambda\sigma})^n q \rangle = (m_0^2)^n \langle \bar{q} q \rangle$  for arbitrary  $n$ .

The section 5 is the **main part** of this paper. We formulate the definition of nonperturbative wave function and give some constraints on it. Finally we model it.

## 2. The QCD vacuum condensates and $\langle \vec{k}_\perp^{2n} \rangle_P$ .

We define the pion pseudoscalar wave function  $\langle 0 | \bar{d}(z) i\gamma_5 u(-z) | \pi(q) \rangle = \frac{-2\langle \bar{q} q \rangle}{f_\pi} \phi_P$  and the corresponding mean values of the quark transverse distribution in the following way:

$$\langle 0 | \bar{d} i\gamma_5 (i\vec{D}_\mu t_\mu)^{2n} u | \pi(q) \rangle = -\frac{2\langle \bar{q} q \rangle}{f_\pi} (-t^2)^n \frac{(2n-1)!!}{(2n)!!} \langle \vec{k}_\perp^{2n} \rangle_P, \quad (2)$$

where  $i\vec{D}_\mu = i\vec{\partial}_\mu + gA_\mu^a \frac{\lambda^a}{2}$  is the covariant derivative and a transverse vector  $t_\mu$  is perpendicular to the hadron momentum  $q_\mu$ . The factor  $\frac{(2n-1)!!}{(2n)!!}$  is related to the integration over  $\phi$  angle in the transverse plane:  $\int d\phi (\cos \phi)^{2n} / \int d\phi = (2n-1)!! / (2n)!!$ .

We removed the common factor  $-\frac{2\langle \bar{q} q \rangle}{f_\pi}$  in order to reproduce the matrix element (1) without derivatives. To find  $\langle \vec{k}_\perp^2 \rangle_P$  we follow the paper [10] and

consider the following matrix element

$$\langle 0 | \bar{d} i \gamma_5 i \vec{D}_\mu i \vec{D}_\nu u | \pi(q) \rangle = -\frac{2\langle \bar{q}q \rangle}{f_\pi} A g_{\mu\nu}, \quad \langle \vec{k}_\perp^2 \rangle_P = -2A, \quad (3)$$

where we dropped off the kinematical structure  $q_\mu q_\nu$  by the reason which will be clear soon. Multiplying (3) by  $g_{\mu\nu}$  and using the equations of motion and PCAC, we get (in the chiral limit,  $m_q \rightarrow 0$ ):

$$\begin{aligned} \langle 0 | \bar{d} i \gamma_5 i \vec{D}_\mu i \vec{D}_\mu u | \pi(q) \rangle &= -\frac{2\langle \bar{q}q \rangle}{f_\pi} 4A = \frac{1}{f_\pi} \langle \bar{q} i g \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q \rangle, \\ \langle \vec{k}_\perp^2 \rangle_P &\simeq \frac{\langle \bar{q} i g \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q \rangle}{4\langle \bar{q}q \rangle} \simeq \frac{m_0^2}{4} \simeq 0.2 \text{GeV}^2, \quad m_0^2 \simeq 0.8 \text{GeV}^2. \end{aligned} \quad (4)$$

where we use the standard value for parameter  $m_0^2$  [6]. It is clear that the skipped term is proportional to  $q_\mu q_\nu g_{\mu\nu} \sim m_\pi^2 \sim m_q$  and gives zero contribution in the chiral limit.

In order to find  $\langle \vec{k}_\perp^4 \rangle_P$  we consider the following matrix element

$$\begin{aligned} \langle 0 | \bar{d} i \gamma_5 i \vec{D}_{\mu_1} i \vec{D}_{\mu_2} i \vec{D}_{\mu_3} i \vec{D}_{\mu_4} u | \pi(q) \rangle &= -\frac{2\langle \bar{q}q \rangle}{f_\pi} \\ &[A g_{\mu_1 \mu_2} g_{\mu_3 \mu_4} + B g_{\mu_1 \mu_3} g_{\mu_2 \mu_4} + C g_{\mu_1 \mu_4} g_{\mu_3 \mu_2}], \end{aligned} \quad (5)$$

where we dropped off the terms which include the  $q_\mu q_\nu$  structure by the reasons mentioned above. The PCAC leads to the following relation

$$\langle 0 | \bar{d} i \gamma_5 i \vec{D}_{\mu_1} i \vec{D}_{\mu_2} i \vec{D}_{\mu_3} i \vec{D}_{\mu_4} u | \pi(q) \rangle = -\frac{2}{f_\pi} \langle \bar{q} P_{\mu_1} P_{\mu_2} P_{\mu_3} P_{\mu_4} q \rangle, \quad (6)$$

where  $P_\mu = i \vec{D}_\mu$  is hermitian operator and condensate  $\langle \bar{d} P_{\mu_1} P_{\mu_2} P_{\mu_3} P_{\mu_4} q \rangle$  can be evaluated in a standard way. The result is:

$$\langle \vec{k}_\perp^4 \rangle_P = \frac{-\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle + 2\langle \bar{q} g^2 G_{\mu\nu} G_{\mu\nu} q \rangle}{12\langle \bar{q}q \rangle}, \quad (7)$$

where  $G_{\mu\nu} = G_{\mu\nu}^a \frac{\lambda^a}{2}$ .

The analogous relations for arbitrary  $n$  are drastically simplified in the limit when the space-time dimension  $d \rightarrow \infty$ . Of course, the real world corresponds to  $d = 4$ , but we expect that the expansion  $1/d^k$  works well enough and errors are of order  $1/d \simeq 25\%$ . Anyhow, we are not going to use this limit for the numerical estimates. Instead, we want to use this limit in order to understand the general structure of the  $\langle \vec{k}_\perp^{2n} \rangle$  (behavior at  $n \rightarrow \infty$ , in particular). In this limit, discarding the small terms related to the creation of the additional  $\bar{q}q$  pair<sup>2</sup>, we get the following simple formula for  $\langle \vec{k}_\perp^{2n} \rangle_P$

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<sup>2</sup>Indeed, the additional terms which appear in this derivation have the form  $[i \vec{D}, \sigma_{\mu\nu} G_{\mu\nu}] \sim (i \vec{D}_\mu G_{\mu\nu}) \gamma_\nu \sim \bar{q} \gamma_\nu \lambda^a q \gamma_\nu$ . It can be checked that these terms are negligible small.

expressed exclusively in terms of condensate  $\langle \bar{q}(ig\sigma_{\mu\nu}G_{\mu\nu})^n q \rangle$ , which is the leading operator in  $d \rightarrow \infty$  limit:

$$\langle \vec{k}_\perp^{2n} \rangle_P \simeq C^n \frac{\langle \bar{q}(ig\sigma_{\mu\nu}G_{\mu\nu}^a \frac{\lambda^a}{2})^n q \rangle}{\langle \bar{q}q \rangle}, \quad d \rightarrow \infty \quad (8)$$

with some constant  $C$ .

Now let us come back to the formula (6). It is clear that the calculation of the mean value of the quark transverse momentum in the pion and the evaluation of some vacuum characteristics are two sides of the same coin.

Let us derive the same relations (6) in a different way. Consider for this purpose the following correlator

$$T_{\nu\ldots\mu_n} = i \int dx e^{iqx} \langle 0 | T \{ \bar{d} i \gamma_5 O_{\mu_1\ldots\mu_n} u(x), \bar{u} \gamma_\nu \gamma_5 d(0) \} | 0 \rangle = q_\nu T_{\{\mu_n\}}(q^2) + \dots (9)$$

with arbitrary local operator  $O_{\mu_1\ldots\mu_n}$ . We extracted the special kinematical structure  $q_\nu T_{\{\mu_n\}}$  in such a way that the tensor  $T_{\{\mu_n\}}$  does not depend on  $q_\mu$ , but depends only on metric  $g_{\mu_i\mu_j}$ . The leading contribution to  $T_{\{\mu_n\}}$  at  $q^2 \rightarrow \infty$  can be easily calculated and is given by

$$T_{\{\mu_n\}}(q^2) = \frac{-2}{q^2} \langle \bar{q} O_{\mu_1\ldots\mu_n} q \rangle + O\left(\frac{1}{q^4}\right). \quad (10)$$

On the other hand, the only particle which can contribute to this correlator is  $\pi$  meson, because  $a_\nu = \bar{u} \gamma_\nu \gamma_5 d$  is conserved in the chiral limit and matrix elements  $\langle 0 | a_\nu | \pi' \rangle$  for the massive pseudoscalar particles are proportional to  $\partial_\nu a_\nu \sim m_q \rightarrow 0$ . At the same time, the axial mesons do not contribute to the kinematical structure we chosen  $q_\nu T_{\{\mu_n\}}$ .

Thus, the whole result (10) comes exclusively from the  $\pi$  meson and any corrections to this statement are proportional to the non-conservation of the current  $\partial_\nu a_\nu \sim m_q$ ; in particular, the next term proportional to  $1/q^4$  comes with the small factor  $m_q$ . Therefore, the correlation function  $T_{\{\mu_n\}}$  looks like a huge peak related to  $\pi$  meson contribution plus some function on  $q^2$  which is nontrivial, but proportional to the small number  $m_q$ . Using the dispersion relation and retaining only  $\pi$  meson contribution to  $Im T_{\{\mu_n\}}$  we recover the formula (6). The great usefulness of this derivation will be appreciated soon, in the analysis of the axial  $wf$ .

### 3. $\langle \vec{k}_\perp^{2n} \rangle_A$ . Leading twist wave function.

We define the pion axial wave function  $if_\pi q_\mu \phi_A = \langle 0 | \bar{d}(z) \gamma_\mu \gamma_5 u(-z) | \pi(q) \rangle$  and the corresponding mean values of the quark transverse distribution in the following way:

$$\langle 0 | \bar{d} \gamma_\nu \gamma_5 (i \vec{D}_\mu t_\mu)^{2n} u | \pi(q) \rangle = if_\pi q_\nu (-t^2)^n \frac{(2n-1)!!}{(2n)!!} \langle \vec{k}_\perp^{2n} \rangle_A. \quad (11)$$

First of all let us show that the calculation of eq. (11) is reduced to the calculation of the matrix elements depending only on  $G_{\mu\nu}$ : We start from the simplest case and consider the following matrix element [10]:

$$\begin{aligned} \langle 0 | \bar{d}\gamma_\nu \gamma_5 \frac{1}{2} [i\vec{D}_{\mu_1} i\vec{D}_{\mu_2} + i\vec{D}_{\mu_2} i\vec{D}_{\mu_1}] u | \pi(q) \rangle = \\ i f_\pi \{ A g_{\mu_1 \mu_2} q_\nu + B [g_{\mu_1 \nu} q_{\mu_2} + g_{\nu \mu_2} q_{\mu_1}] \} \end{aligned} \quad (12)$$

From the definition of  $\langle \vec{k}_\perp^2 \rangle_A$  it is clear, that  $\langle \vec{k}_\perp^2 \rangle_A = -2A$ . At the same time, multiplying (12) by metric tensor and using equations of motion, the constant  $A$  can be expressed in terms of the following matrix element:

$$\langle 0 | \bar{d}\gamma_\nu \gamma_5 \left( \frac{-ig}{2} \right) \sigma_{\lambda\sigma} G_{\lambda\sigma} u | \pi(q) \rangle = \frac{18}{5} i f_\pi q_\nu A \quad (13)$$

In order to express  $\langle \vec{k}_\perp^4 \rangle_A$  in the same terms, we consider the following matrix element

$$\langle 0 | \bar{d}\gamma_\mu \gamma_5 \frac{1}{4!} \sum_{\{\mu_1, \mu_2, \mu_3, \mu_4\}} (i\vec{D}_{\mu_1} i\vec{D}_{\mu_2} i\vec{D}_{\mu_3} i\vec{D}_{\mu_4}) u | \pi(q) \rangle. \quad (14)$$

By the reason mentioned above, we have to consider on the right hand side of this equation the most general kinematical structure which includes only the first power of  $q_\mu$  and which is symmetric under  $\mu_1, \mu_2, \mu_3, \mu_4$  permutations; the rest indexes are cared by the metric tensor  $g_{\mu\nu}$ . Multiplying (14) by  $g_{\mu\nu}$ , using the equation of motion and skipping all small terms which are related to creation of the additional  $\bar{q}q$  pair (for justification, see below), we arrive to the following result:

$$\begin{aligned} i q_\rho f_\pi \langle \vec{k}_\perp^4 \rangle_A = \frac{3}{8} \langle 0 | \bar{d}\gamma_\rho \gamma_5 \left( \frac{-ig}{2} \right) \sigma_{\lambda\sigma} G_{\lambda\sigma}^2 u | \pi \rangle + \\ \frac{3}{16} \langle 0 | \bar{d}\gamma_\rho \gamma_5 g^2 G_{\mu\nu} G_{\mu\nu} u | \pi \rangle + \frac{1}{72} \langle 0 | \bar{d}\gamma_\mu \gamma_5 g^2 (G_{\nu\rho} G_{\mu\nu} + G_{\mu\nu} G_{\nu\rho}) u | \pi \rangle. \end{aligned} \quad (15)$$

Again, the problem is reduced to the calculation of some matrix elements, which can be estimated by the method, described in the previous section and which we are going to use now.

In order to calculate the  $\langle \vec{k}_\perp^2 \rangle_A$  let us consider the following correlator (compare with (9)):

$$i \int dx e^{iqx} \langle 0 | T \{ \bar{d}\gamma_\mu \gamma_5 i g \sigma_{\lambda\sigma} G_{\lambda\sigma} u(x), \bar{u} i \gamma_5 d(0) \} | 0 \rangle = q_\mu T(q^2) + \dots \quad (16)$$

In comparison with the previous case, the current  $J_\mu = \bar{d}\gamma_\mu \gamma_5 i g \sigma_{\lambda\sigma} G_{\lambda\sigma} u$  is not conserved even in the chiral limit, but it is “almost” conserved  $\partial_\mu J_\mu \sim (i\vec{D}_\mu G_{\mu\nu}) \sim \bar{q}\gamma_\nu q$  in a sense, that the non-conservation is small and related

to the production of the additional  $\bar{q}q$  pair. From the point of view of the expansion  $T(q^2)$  at large  $q^2$  it means that the main contribution is given by

$$T(q^2) = \frac{i}{q^2} \langle \bar{q} i g \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle, \quad (17)$$

like in the previous case (10); the corrections to this formula are suppressed by the loop factor  $\frac{\alpha_s}{\pi} \simeq 0.1$ . We neglect them in the following. Using a dispersion relation and retaining the  $\pi$  meson contribution only<sup>3</sup>, one gets [10]:

$$\langle \vec{k}_\perp^2 \rangle_A = \frac{5}{36} \frac{\langle \bar{q} i g \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2} q \rangle}{\langle \bar{q} q \rangle} \simeq \frac{5m_0^2}{36} \simeq 0.1 GeV^2. \quad (18)$$

The same procedure can be applied for any operator whose matrix element we are interested in. The result for  $\langle \vec{k}_\perp^4 \rangle_A$  is:

$$\langle \vec{k}_\perp^4 \rangle_A = \frac{1}{8} \left\{ \frac{-3 \langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle}{4 \langle \bar{q} q \rangle} + \frac{13 \langle \bar{q} g^2 G_{\mu\nu} G_{\mu\nu} q \rangle}{9 \langle \bar{q} q \rangle} \right\}, \quad (19)$$

The problem, like in the previous case is reduced to the analysis of the mixed vacuum condensates.

#### 4. Analysis of the condensates.

Let me, first of all, formulate the method we follow in this section in order to estimate the higher dimensional condensates. The idea is as follows. Consider the correlation function  $T_\mu = i \int dx e^{iqx} \langle T \{ J_\mu(x), J(0) \} \rangle$  at large  $q^2$ . If the currents  $J_\mu, J$  are chosen in such a way, that in the chiral limit the perturbative contribution is zero and the current  $J_\mu$  is “almost” conserved (in a sense of the previous section), we end up with the leading (at  $q^2 \rightarrow \infty$ ) contribution in the form  $\frac{\langle O \rangle}{q^2}$ , like (17), plus some nontrivial function, but with small coefficient, like  $\frac{\alpha_s}{\pi}$ , or  $m_q$ , in front of it. Besides that, if we know the  $\pi$ -meson matrix elements  $\langle 0 | J_\mu | \pi \rangle, \langle 0 | J | \pi \rangle$  exactly, we can find the condensate  $\langle O \rangle$  by collecting all leading terms proportional to  $1/q^2$ . To convince the reader in correctness of this method, we will explicitly calculate the loop corrections as well in the example which follows.

In order to calculate the condensate  $\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle$ , let us consider the following correlator:

$$i \int dx e^{iqx} \langle 0 | T \{ \bar{d} \gamma_\rho \gamma_5 i g \sigma_{\lambda\sigma} G_{\lambda\sigma} u(x), \bar{u} \gamma_5 (i g \sigma_{\mu\nu} G_{\mu\nu}) d(0) \} | 0 \rangle = q_\rho T(q^2) + \dots \quad (20)$$

The leading contribution is determined by the following condensate, which is unknown:

$$T(q^2) = \frac{1}{q^2} \langle \bar{q} (i g \sigma_{\lambda\sigma} G_{\lambda\sigma})^2 q \rangle. \quad (21)$$

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<sup>3</sup>The spectral density  $ImT(s)$  falls off quickly at large  $s$  in the chiral limit. This is an additional justification for the keeping only  $\pi$  meson contribution.

At the same time, both  $\pi$  meson matrix elements which enter to this correlator are known – one of them:  $\langle 0 | \bar{d} \gamma_\rho \gamma_5 i g \sigma_{\lambda\sigma} G_{\lambda\sigma} u(x) | \pi \rangle$  is from (13) and another one:  $\langle 0 | \bar{u} \gamma_5 (i g \sigma_{\mu\nu} G_{\mu\nu}) d(0) | \pi \rangle$  is from (4) and can be found from PCAC. Thus, we have finally the following very important equation:

$$\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle = - \frac{\langle \bar{q} i g \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle^2}{\langle \bar{q} q \rangle} \quad (22)$$

Before we proceed with the numerical estimates, let me briefly formulate the result of the loop calculation for the correlator under consideration. After the “borelization” procedure [5] the leading loop contribution to the  $T(M^2)$  is given by:  $T(M^2) = -\frac{\alpha_s}{12\pi} M^2 \langle \bar{q} q \rangle$ , where  $M^2$  is Borel parameter which is determined by the next power corrections (which have the same small factor  $\alpha_s/\pi$ ) and usually runs in the region  $M^2 \simeq 0.6 \div 1 \text{ GeV}^2$ . This term is 30 times less than both contributions: (21) and  $\pi$  meson residue (22). This rate gives us some intuition about magnitude of the loop corrections to the eq.(22). We neglect them in the rest of the paper.

The factorization prescription for the condensate (21) leads to the following formula:

$$\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle = -\frac{K}{3} \langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle \langle \bar{q} q \rangle, \quad K = \frac{3m_0^4}{\langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle} \simeq 3, \quad (23)$$

where we have introduced the coefficient of nonfactorizability  $K$  ( $K = 1$  if the factorization would work). The last equation follows from (22) and actually demonstrates that the factorization does not work.

Few comments are in order. First of all, let us note that this result (the violation of factorization by a factor 3) is in a full agreement with the (absolutely independent) analysis [11] of the mixed vacuum condensates of the dimension seven. The result was based on the analysis of the heavy-light quark system and supports the present consideration. Besides that, let us note, that the VEV which appeared in the analysis [11] was actually some combination of condensates  $\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle$  and  $\langle \bar{q} g^2 G_{\mu\nu}^a G_{\mu\nu}^a q \rangle$ . We found for that combination more or less the same coefficient for the nonfactorizability  $K \simeq 3$ . So, for the numerical estimates it is natural to assume that the factor  $K \simeq 3$  is the universal one for all condensates of this kind.

As the second remark, we note that the phenomenon of non-factorizability for the mixed condensates is not a big surprise, because we faced the analogous phenomenon early [12] for the four-fermion condensates with an “exotic” Lorentz structure.

As the last remark, and probably, the most important one, we want to emphasize that the eq.(22) can be considered as the recurrent relation between  $\langle \bar{q} i g \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle$  and  $\langle \bar{q} g^2 \sigma_{\mu\nu} G_{\mu\nu} \sigma_{\lambda\sigma} G_{\lambda\sigma} q \rangle$  with the dimensional coefficient  $m_0^2$ . We can repeat this procedure for arbitrary  $n$  with result:

$$\langle \bar{q} (i g \sigma_{\lambda\sigma} G_{\lambda\sigma})^n q \rangle = (m_0^2)^n \langle \bar{q} q \rangle, \quad (24)$$



which we expect is to be correct one with a high accuracy  $\sim \alpha_s/\pi^4$ . The formula (24) is the **main result** of this section. First of all it demonstrates a very simple relation between condensates and gives very important phenomenological information for model constructors of the QCD vacuum. Secondly, bearing in mind that these condensates are directly related to the moments  $\langle \vec{k}_\perp^{2n} \rangle$ , the formula (24) gives nontrivial information about nonperturbative pion wave function  $\psi(\vec{k}_\perp^2, x)$ .

### 5. Constraints on the nonperturbative wave function $\psi_A(\vec{k}_\perp^2, x)$ .

Now we are going to consider some applications of the obtained results. First of all let us consider the pseudoscalar  $wf$  and its moments. From eqs. (4,7) we have the following ratio for  $\langle \vec{k}_\perp^4 \rangle_P / \langle \vec{k}_\perp^2 \rangle_P^2$ :

$$\frac{\langle \vec{k}_\perp^4 \rangle_P}{\langle \vec{k}_\perp^2 \rangle_P^2} = K \frac{8\langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle}{9m_0^4} \simeq 0.6K \simeq 2, \quad (25)$$

where we assumed the universality for the nonfactorizability factor  $K \simeq 3$ , as explained above. Let us remark, that we know the condensate (22), which enters to the expression for  $\langle \vec{k}_\perp^4 \rangle_P$  “almost” exactly. However, we do not possess such precise information for the second VEV which contributes to  $\langle \vec{k}_\perp^4 \rangle_P$  with the same weight. In spite of this fact we expect a good accuracy for the ratio, because both contributions go with the *same* signs, and because some independent combination of these VEVs leads to the same coefficient  $K \simeq 3$ , see above. From the pure theoretical point of view, by considering the limit  $d \rightarrow \infty$ , we can argue that there is not any cancellation between different terms (most dangerous thing which could happen!).

This ratio actually is some expression of the fluctuations of the momentum. The result (25) demonstrates that the distribution function for the pseudoscalar function is rather compact in the momentum space in a big contrast with the analogous distribution for the leading twist wave function.

For the axial  $wf$  from the formulae (18,19) we have the following ratio (instead of eq.(25)):

$$\frac{\langle \vec{k}_\perp^4 \rangle_A}{\langle \vec{k}_\perp^2 \rangle_A^2} \simeq 3K \frac{\langle g^2 G_{\mu\nu}^a G_{\mu\nu}^a \rangle}{m_0^4} \simeq 5 \div 7, \quad (26)$$

which tells us that the fluctuations of the transverse momentum are much larger for the axial  $wf$  than for the pseudoscalar  $wf$ <sup>5</sup>.

<sup>4</sup>The analogous result for the  $\langle \bar{q}(G_{\lambda\sigma})^n q \rangle$  looks as follows:  $\langle \bar{q}(G_{\lambda\sigma}^a G_{\lambda\sigma}^a)^n q \rangle = (m_1^4)^n \langle \bar{q}q \rangle$ , but with unknown coefficient  $m_1^4$ .

<sup>5</sup>Let us note, that the same ratio was examined in the recent preprint [13] by the method of [9] with the result  $\frac{\langle \vec{k}_\perp^4 \rangle_A}{\langle \vec{k}_\perp^2 \rangle_A^2} = 9$  which numerically is not far away from our estimates (26). However from our opinion, the corresponding sum rules are not suitable for the calculations of  $\langle \vec{k}_\perp^n \rangle_A$ . We see at least few reasons for that. First of all, the

It is interesting to note, that the analysis of the distribution function for the longitudinal momentum  $\langle \xi^{2n} \rangle$  [9] shows the same phenomenon: deviation of longitudinal moments from their asymptotic values is large for the axial  $wf$  and very modest for the pseudoscalar  $wf$ .

In principle we are ready to model  $\psi(\vec{k}_\perp^2, x)$ . However, as is known, the knowledge of a finite number of moments is not sufficient to completely determine the  $\psi(\vec{k}_\perp^2, x)$ . The behavior of the asymptotically distant terms is a very important thing as well. Let me remind you of an example of the well-known case for the longitudinal distribution which determined by the  $\phi(\xi)$ . If  $\langle \xi^m \rangle$  would behave as  $\sim 1/m$  at large  $m$ , it would mean that the integrand  $\int_{-1}^1 d\xi \xi^m \phi(\xi) \sim 1/m$  implying that  $\phi(\xi \rightarrow \pm 1) \rightarrow \text{constant}$ . At the same time, the very likely assumption that the  $\pi$ -meson fills a finite duality interval in the dispersion relation at any  $m$  means that  $\int_{-1}^1 d\xi \xi^m \phi(\xi) \sim 1/m^2$  [2]. It unambiguously implies the behavior

•1  $\phi(\xi \rightarrow \pm 1) \rightarrow (1 - \xi^2).$

A simple model wave function which meets this requirement as well as possesses a large value of the lowest moment,  $\langle \xi^2 \rangle \simeq 0.4$  has been proposed in ref. [9]:

$$\phi(\xi) = \frac{15}{4}(1 - \xi^2)\xi^2, \quad \langle \xi^0 \rangle = 1, \quad \langle \xi^2 \rangle = 0.43 \quad (27)$$

We want to emphasize that the “endpoint” behavior (which corresponds to asymptotically distant terms) was crucial in this analysis.

In terms of  $\psi(\vec{k}_\perp^2, \xi = 2x - 1)$  this constraints can be rewritten in the following form:  $\int d\vec{k}_\perp^2 \psi(\vec{k}_\perp^2, \xi \rightarrow \pm 1) \sim (1 - \xi^2)$ . The analogous assumption, that the  $\pi$ -meson fills a finite duality interval in the corresponding dispersion relation at any  $n$ , where  $n$  is related to  $n$ - transverse moment  $\langle \vec{k}_\perp^{2n} \rangle$  gives the following constraint:

•2  $\int d\vec{k}_\perp^2 \vec{k}_\perp^{2n} \psi(\vec{k}_\perp^2, \xi \rightarrow \pm 1) \sim (1 - \xi^2)^{n+1} .$

This constraint is extremely important and implies that the  $\vec{k}_\perp^2$  dependence of the  $\psi(\vec{k}_\perp^2, \xi = 2x - 1)$  comes **exclusively in the combination**  $\vec{k}_\perp^2/(1 - \xi^2)$  at  $\xi \rightarrow \pm 1$ . The byproduct of this constraint can be formulated as follows. The standard assumption on factorizability of the  $\psi(\vec{k}_\perp^2, \xi) = \psi(\vec{k}_\perp^2)\phi(\xi)$  **does contradict** to the very general property of the theory formulated above.

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corresponding spectral density increases very quickly  $\sim s^n$  with  $s$  and there is no reason for  $\pi$  meson saturation. As a consequence of it, there is a very strong dependence on  $S_0$  which is far away ( $S_0 \simeq 0.4 GeV^2$ ) from the standard value  $S_0 \simeq 1 GeV^2$ . By some kinematical reasons, the 4-quark condensate, which used to be the most important one, does not contribute. It clearly means, that the next power corrections, proportional to  $\langle \bar{q}(ig\sigma_{\lambda\sigma}G_{\lambda\sigma})^n q \rangle \langle \bar{q}q \rangle$  become very important. Thus, our point is that the corresponding sum rules do not work and moments  $\langle \vec{k}_\perp^{2n} \rangle$  can not be extracted in such way.

The next constraints come from the calculation  $\langle \vec{k}_\perp^2 \rangle_A$ , (18) and  $\langle \vec{k}_\perp^4 \rangle$ , (26):

$$\bullet 3 \int d\vec{k}_\perp^2 \int_{-1}^1 d\xi \psi(\vec{k}_\perp^2, \xi) = 1, \langle \vec{k}_\perp^2 \rangle = \frac{5m_0^2}{36} \simeq 0.1 \text{GeV}^2, \quad \langle \vec{k}_\perp^4 \rangle \gg \langle \vec{k}_\perp^2 \rangle^2.$$

These constraints actually give the general scale of the  $\psi(\vec{k}_\perp^2, \xi)$ . Besides that, the large fluctuations of the momentum mean, first of all, that the distribution of the transverse quark momentum is wide and it is not concentrated about  $\vec{k}_\perp^2 \simeq 0$ .

Our last constraint comes from the analysis of the asymptotically distant terms. We do not know the behavior of the asymptotically large moments  $\langle \vec{k}_\perp^{2n} \rangle$  at  $n \rightarrow \infty$  exactly. But we do know the dependence on  $n$  in the limit when the space-time dimension  $d \rightarrow \infty$ , (8)<sup>6</sup>. We expect (and this is the main assumption) that in the real world the functional dependence will not be changed. Thus, for the asymptotically large  $n$  we expect the following behavior:

$$\bullet 4 \quad \langle \vec{k}_\perp^{2n} \rangle \Rightarrow \frac{\langle \bar{q}(ig\sigma_{\mu\nu}G_{\mu\nu}^a \frac{\lambda^a}{2})^n q \rangle}{\langle \bar{q}q \rangle} \sim (m_0^2)^n,$$

where at the last stage we have used the eq.(24), which is suppose to be valid for the arbitrary  $n$ . It is very important that the right hand side of this equation is constant and not zero. We are not going to use this constant in our model as input parameter; the only fact we need, that this is not zero. Let us note, that any mild function on  $n$ , like  $1/n$  or even  $\exp(n)$  on the right hand side can not be ruled out. The only effect it brings, is some rescale of the dimensional parameter:  $m_0^2 \rightarrow \tilde{m}_0^2 = \sqrt[n]{n} m_0^2 = m_0^2$  or  $m_0^2 \rightarrow \tilde{m}_0^2 = e m_0^2$ , which anyhow, is unknown. The same constraint can be obtained from the dispersion relation with the assumption that the  $\pi$  meson fills a finite duality interval  $S_0$  at any  $n$ , like in the analysis ( $\bullet 1$ ). In this case, instead of ( $\bullet 4$ ) we get

$$f_\pi^2 \langle \vec{k}_\perp^{2n} \rangle \Rightarrow \frac{3S_0^{n+1}(2n+2)!!}{8\pi^2(2n+3)!!} \sim \frac{S_0^{n+1}}{n},$$

which can be reduced to the previous one.

Very important consequence of this constraint can be easily seen if we rewrite it in the following form:

$$\bullet 4 \quad \int d\vec{k}_\perp^2 \psi\left(\frac{\vec{k}_\perp^2}{1-\xi^2}, \xi\right) (\vec{k}_\perp^2)^n \Rightarrow 1,$$

which means that the  $\psi(\frac{\vec{k}_\perp^2}{1-\xi^2}) \Rightarrow \delta(\frac{\vec{k}_\perp^2}{1-\xi^2} - S)$  for sufficiently large  $\vec{k}_\perp^2$ . We have to pause here in order to explain the general idea of the Wilson operator

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<sup>6</sup>We derived this formula for pseudoscalar function only, but absolutely the same formula takes place for the axial  $wf$  as well. The derivation is the same.

expansion (OPE) in the given context and the term “sufficiently large  $\vec{k}_\perp^2$ ” in particular.

As is known, all VEVs within OPE are defined in such a way, that all gluon’s and quark’s virtualities, smaller than some parameter  $\mu$  (point of normalization) are hidden in the definition of the “nonperturbative vacuum matrix elements”. All virtualities larger than that should be taken into account perturbatively, see [6] for detail discussion of this problem. At the same time, from the exact PCAC relations, like (4) or (7) it is clear that moments of the wave functions are related to condensates, defined as explained above. Thus, all transverse moments are defined in the same way as condensates do. This is, actually, the **definition of the nonperturbative wave function**  $\psi(\vec{k}_\perp^2, \xi)$ , through its moments which can be expressed in terms of nonperturbative vacuum condensates.

Now it clear, what we mean by term “sufficiently large  $\vec{k}_\perp^2$ ”. By that we mean the largest virtuality which have been taken into account in construction of  $\psi(\vec{k}_\perp^2, \xi)$ , or (what is the same) in the definition of condensates. The corresponding dimensional parameter can be expressed in terms of some condensate (see below) and it depends on  $\mu$  only logarithmically, like condensates do. The important consequence of the definition can be formulated in the following way: The nonperturbative  $wf$  even for sufficiently large  $\vec{k}_\perp^2$  does not behave like  $1/\vec{k}_\perp^2$  as is frequently assumed. In order to understand this statement let us imagine that we calculate the gluon condensate  $\langle G_{\mu\nu}^2 \rangle$ . It is clear that we will not substitute the gluon propagator in the form  $1/k^2$  in this calculation, because, by definition, the perturbative contribution should be subtracted in the calculation of the nonperturbative condensate. Have in mind that our definition of the nonperturbative  $wf$  is formulated in terms of the vacuum condensates, it is clear that the same statement is concerned to the nonperturbative wave function as well and its behavior has nothing to do with  $1/\vec{k}_\perp^2$ .

We want to model the simplest version of the wave function which meets all requirements formulated above. First of all, as was explained, the  $\vec{k}_\perp^2$  dependence comes only in the combination  $\psi(\frac{\vec{k}_\perp^2}{1-\xi^2})$  in order to meet the requirement (•2). In order to satisfy the constraint (•4), we have to assume that at sufficiently large  $\vec{k}_\perp^2$  we have a  $\delta(\frac{\vec{k}_\perp^2}{1-\xi^2} - S)$  -like function, whose moments correctly reproduce the constant on the right hand side of the equation; the  $S$  is some input dimensional parameter, which will be expressed in terms of  $\langle \vec{k}_\perp^2 \rangle$ .

Our next step is to satisfy the constraint (•3). It is clear that the  $\delta$  function proposed above does not provide a noticeable fluctuations of the momentum. In order to meet this requirement, we have to spread out the distribution function between  $\vec{k}_\perp^2 \sim 0$  and  $\vec{k}_\perp^2 \sim S$  in such a way, that the overall area will be the same, but moments should satisfy to this requirement. In principle, it can be done in arbitrary way. The simplest way to make the

$wf$  wider is to put another  $\delta$  function at  $\vec{k}_\perp^2 = 0$ . With these remarks in mind we propose the following "two-hump" (*again!*) nonperturbative wave function which meets all requirements discussed above:

$$\psi(\vec{k}_\perp^2, \xi) = [A\delta(\frac{\vec{k}_\perp^2}{1-\xi^2} - S) + B\delta(\frac{\vec{k}_\perp^2}{1-\xi^2})][g(\xi^2 - \frac{1}{5}) + \frac{1}{5}]$$

$$\int d\vec{k}_\perp^2 \int_{-1}^1 d\xi \psi(\vec{k}_\perp^2, \xi) = 1, \quad A + B = \frac{15}{4} \quad (28)$$

$$\phi(\xi) \equiv \int d\vec{k}_\perp^2 \psi(\vec{k}_\perp^2, \xi) = \frac{15}{4}(1 - \xi^2)[g(\xi^2 - \frac{1}{5}) + \frac{1}{5}]$$

$$\langle \vec{k}_\perp^4 \rangle \simeq 5\langle \vec{k}_\perp^2 \rangle^2 \Rightarrow A = 7/8, \quad g = 1, \quad \langle \vec{k}_\perp^2 \rangle = \frac{5m_0^2}{36} \Rightarrow S = \frac{15}{2}\langle \vec{k}_\perp^2 \rangle \simeq 0.8 GeV^2$$

Few comments are in order. We put the common factor  $[g(\xi^2 - \frac{1}{5}) + \frac{1}{5}]$  in the front of the formula in order to reproduce the light cone  $\phi(\xi)$  function with arbitrary  $\langle \xi^2 \rangle$ . For  $g = 0$  it corresponds to the asymptotic  $wf$ :  $\phi(\xi) = 3/4(1 - \xi^2)$ . For  $g = 1$  we reproduce the  $\phi(\xi)_{CZ} = 15/4(1 - \xi^2)\xi^2$  with  $\langle \xi^2 \rangle \simeq 0.43$ . We still believe in large value for the moment  $\langle \xi^2 \rangle$  in spite of the criticism from the ref.[14], who found much less value for  $\langle \xi^2 \rangle$ . This is not the place to discuss this question in more detail, but I want to make a comment that there are few questions to be answered ( theoretical, as well as phenomenological ones) before the approach advocated by authors of ref.[14] can be considered as is well defined and based on solid ground. At the same time, the people who prefer to use a smaller value for  $\langle \xi^2 \rangle$ , can make their own choice by changing the parameter  $g$  in the eq.(28). Let me remind the relation  $\langle \xi^2 \rangle = 1/5 + 8g/35$  between  $\langle \xi^2 \rangle$  and parameter  $g$  from the formula (28) for doing so. It will not spoil the constraints discussed above.

Let me emphasize, that the formula (28) is not destined for the precise fitting of the experimental data and it is given as illustration only. By the physical reasons it is clear that the  $\delta$  functions should be spread out in some way. This procedure is definitely model dependent. We only want to attract attention on the asymmetric form of the  $wf$  in the  $\vec{k}_\perp$  space. This qualitative property of the  $wf$  comes from the large magnitude of the ratio  $r = \langle \vec{k}_\perp^4 \rangle / \langle \vec{k}_\perp^2 \rangle^2$ .

As the last remark, let me point out, that the description of exclusive reactions at experimentally accessible momentum transfers ( $1 GeV^2 < Q^2 < 30 GeV^2$ ) and the problem of extracting some information about nonperturbative  $wf$  are two different problems. It is was advocated by Radyushkin and collaborators [15], [16] and Isgur, Llewellyn Smith [17] for a long time that the asymptotically leading contributions do not describe the experimental data at moderate  $Q^2$ , for recent review of this subject, see [18]. Some

confirmation of this statement came recently from [8], where it was shown that the inclusion of the intrinsic  $\vec{k}_\perp^2$ -dependence and Sudakov suppression leads to the self-consistent calculation, but the obtained magnitude is too small with respect to the data even if  $\phi_{CZ}$  asymmetric function is used. It is very likely, that the "soft" contributions play an important role in this region.

It turns out, that the very unusual shape of the  $\psi(\vec{k}_\perp^2, \xi)$  described above supports this idea and can **imitate** the behavior of the asymptotically leading contribution at the intermediate momentum transfer. Such a mechanism, if it is correct, would be an explanation of the phenomenological success of the dimensional counting rules at a very modest  $Q^2$ . We are going to discuss this question in more detail somewhere else [19].

## 6. Conclusion.

Let me formulate the main results of this paper.

First of all, we used the standard definition of the nonperturbative  $wf$  through its moments and we expressed these moments in terms of the vacuum condensates. From this construction it becomes clear that the definition of the vacuum condensates (within OPE), with subtraction of the perturbative contributions, and the definition of the nonperturbative  $wf$  is one and the same problem.

Secondly, we formulated some constraints on the nonperturbative  $wf$ ; they have very general origin, and thus, they should be satisfied for any model. There is no reason to repeat these constraints (●1 – ●4) from the previous section again; the only comment we want to make here is the following: the standard assumption on factorizability of the  $wf$ :  $\psi(\vec{k}_\perp^2, x) = \psi(\vec{k}_\perp^2)\phi(x)$  can not be matched with these constraints.

The last result which deserves to be mentioned here and which probably is interesting by its own, regardless to the question of nonperturbative  $wf$ , looks as follows  $\langle \bar{q}(ig\sigma_{\lambda\sigma}G_{\lambda\sigma})^n q \rangle = (m_0^2)^n \langle \bar{q}q \rangle$ . I am not aware of any model of the QCD vacuum, which satisfies to this "almost" exact relation between mixed vacuum condensates.

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<sup>7</sup> The correspondence is reached only in the limit  $d \rightarrow \infty$

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